Pressure Recovery in Rectangular Constant Area Supersonic Diffusers

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Experimental results on the performance of a supersonic diffuser consisting of a constant area duct with rectangular cross section are reported. The influence of the Reynolds number ($4\times10^4 < Re_D < 4\times10^5$), Mach number ($1.76<M<2.88$), a boundary-layer parameter ($0.04<\delta^*/D<0.20$), and the duct length ($L/D=15, 30, 45, 60$) on the optimum pressure recovery with a clean test section have been investigated. A unified viewpoint based on the variables of the experiment is developed and certain results in the literature together with the current findings are correlated. These graphical interpolations appear to be new and as a useful tool to predict diffuser performance, they simplify the design of new diffusers of the straight duct type.

Nomenclature

D = hydraulic diameter (equal to 4 times area divided by circumference). Nozzle exit area (1.91×0.95) cm^2, $D=1.27$ cm
L = duct length, $L/D=15, 30, 45, 60$
M = Mach number $(M=v/a)$
P = static pressure
P_t = pitot pressure
Δp = diffuser pressure recovery $(\Delta p = p_{max} - p_{initial})$†
Re_D = Reynolds number based on hydraulic diameter and test section freestream parameters. $(Re_D = (v\rho \cdot D)/\mu)$
T = temperature
v = flow speed
x = axial distance, at nozzle exit $x=0$
Δx = length of pressure recovery zone (from start of pressure rise to maximum pressure)†
δ = boundary-layer thickness
δ^* = boundary-layer displacement thickness
ρ = density
μ = dynamic viscosity

Subscripts

0 = reservior, nozzle supply
1 = nozzle end
2 = diffuser end

Introduction

THE problem of pressure losses associated with supersonic diffusers became technologically important long ago in view of the substantial power consumption of large supersonic and hypersonic wind tunnels. Such engineering problems spurred various investigations[1-5] (for more complete references see Johnson and Wu[6]), which became available in the early 1950's. The main results of these experiments are that fixed geometry diffusers yield approximately the normal shock pressure recovery computed at the test section Mach number, and variable geometry diffusers give about 1.5 times and at hypersonic velocities even 2 or more times normal shock pressure recovery. Because pressure losses in normal shocks are high, especially at higher Mach numbers, these diffusers still represent inefficient devices for pressure recovery. In view of the complex flow patterns observed in diffusers, it was moreover found that the problem was difficult to tackle theoretically, thus leaving us with little general understanding on diffusers resulting from such work. Eventually, large losses had to be accepted, especially because of additional often unpredictable pressure losses due to the presence of models in the wind tunnel test section. In fact the design of high-efficiency supersonic diffusers can only be based on little general data at this time. This lack of general knowledge became particularly acute if new devices were to be designed. For example, gasdynamic lasers may have duct geometries previously not tested (e.g., Russel[7]) and no models are installed in the test section. Similar problems are found in engines, rotating machinery, etc. For such reasons new interest in supersonic diffuser performance has arisen.[8-11] Thus, the present report aims at adding to our knowledge of the simplest supersonic diffuser, the rectangular constant area duct with a systematic investigation of the diffuser length and performance in a range of Mach numbers, Reynolds numbers, and boundary-layer parameters, without models being present.

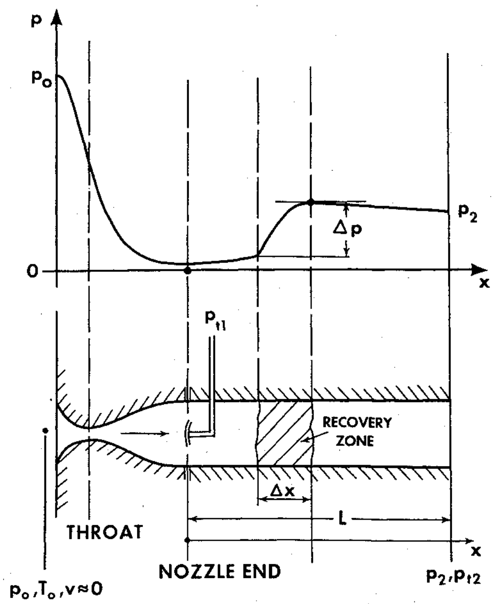

Fig. 1 Symbols and nomenclature.

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Index categories: Nozzle and Channel Flow; Supersonic and Hypersonic Flow.

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†See also Fig. 1.

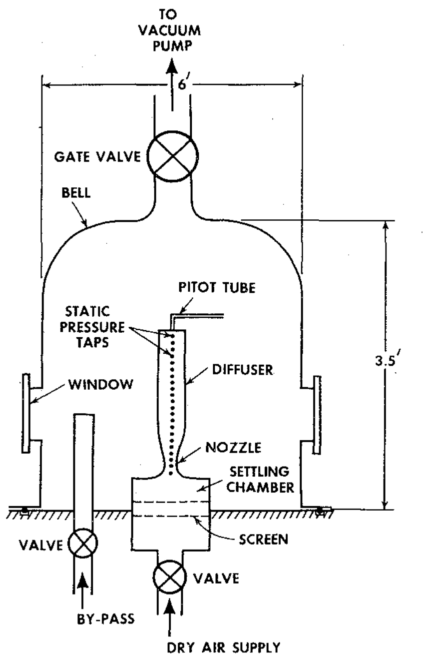

Fig. 2 Sketch of wind tunnel.

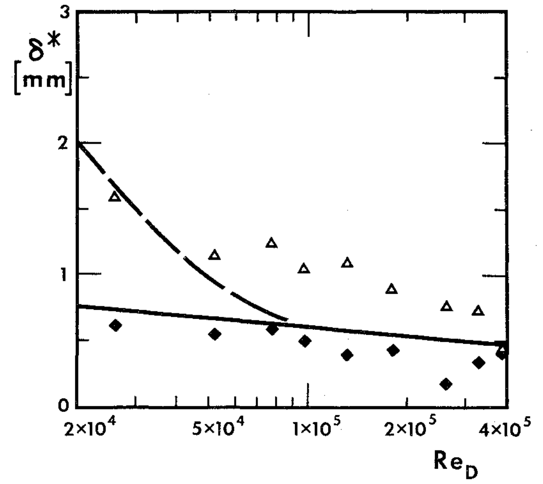

Fig. 3 The boundary-layer displacement thickness δ^* in the nozzle exit plane as a function of the Reynolds number based on the hydraulic diameter Re_D. The measurements are made with the constant area diffuser in place. △ Probe measurements perpendicular to the center of the large sidewall (1.91 cm). ▲ Probe measurements perpendicular to the center of the small wall (0.95 cm). --- Average value deduced from the observed effective area of the nozzle. —— Predictions after Tuckers method.[14]

Experimental Apparatus and Test Procedure

A continuously operating two-dimensional supersonic nozzle was used (throat to exit length 7.62 cm), designed with the method of characteristics for a uniform $M=3.0$ flow (exit cross section (1.91×0.95) cm^2, corresponding to a hydraulic diameter of $D=1.27$ cm). Constant area ducts (waveguide tubes) of lengths $L/D=15$, 30, 45, and 60 were attached flush to the nozzle. From the duct exit the flow was dumped into a large bell-like chamber, (see Fig. 2) in which the entire apparatus was mounted, taking advantage of an existent facility.[12] Connected to a constant volume vacuum pump, the pressure in the chamber, i.e., the back pressure of the diffuser, could be controlled by a gate valve and a by-pass system. By means of a pressure control valve, the supply pressure of the oil-free compressed dry air with $T_0\approx 300$ K, which was drawn from two, continuously pumped storage tanks at 8 atm abs pressure ($V=17$ m^3) could be controlled in the range from 100 to 3050 Torr (0.13-4 atm). This supply pressure, static pressures along nozzle and diffuser, and the pitot pressure in the diffuser exit plane were measured. The static pressure taps with an inner diameter of roughly 0.1 cm were spaced at $1D$ intervals along the centerline of the flat nozzle wall and continued in the centerline of the larger (i.e., 1.91 cm) duct wall. For the longer ducts the spacing of the pressure taps was increased. With the scanning valves the static pressure could be measured at 24 taps in a given experiment. The pitot pressure in the duct exit plane was usually measured in the center of the flow. Yet the pitot probe was also traversed along the y- and z-axis (symmetry axis) of the exit cross section.

In a typical experiment, $p_{t0}=p_0$ was set and kept constant after the tunnel was started. Then the back pressure was slowly increased, thereby moving the pressure recovery zone upstream in the duct. The maximum back pressure with which undisturbed nozzle flow could be sustained, as well as the pressure distribution along the flow direction were then recorded. At an essentially constant supply temperature (300 K), a change in the supply pressure proportionally varies the Reynolds number Re_D. Since the nozzle thus was not used at a fixed Reynolds number, no boundary-layer corrections were applied. Measurements with a static pressure probe traversing the centerline of the nozzle and the static pressure measurements along the nozzle sidewall revealed that the supersonic flow was smooth. By comparing theoretical and experimental pressure distributions along the nozzle, the effective area and in turn an estimated average boundary-layer displacement thickness δ^* can be deduced.‡ From Tucker's method,[14] the fully developed turbulent boundary-layer displacement thickness in the nozzle δ^* can also be computed from the initially known design pressure, assuming an insulated wall and the turbulent boundary layer as beginning at the nozzle throat. A comparison of calculated and experimental results on δ^* at the nozzle exit is shown in Fig. 3. In addition to theory and effective area estimates, probe measurements of the displacement thickness are also shown. Here a traversing flattened pitot pressure probe of outside dimensions (0.203×0.073) cm^2 was moved perpendicular to the middle of the larger and smaller sidewall of the rectangular flow channel. With the shortest diffuser ($L/D=15$) attached to the nozzle, the probe protruded 2.6 cm upstream, the tip being in the nozzle exit plane. Assuming a constant static pressure distribution over the duct cross section, the velocity profile can be deduced from these measurements. With a graphical integration of the velocity profiles, the boundary-layer displacement thickness can then be determined. As seen in Fig. 3, the displacement thickness determined in this manner is larger on the larger sidewall than on the smaller wall, which in our case coincides with the contoured nozzle surface. The two values, however, bracket the displacement thickness as determined from the comparison of ideal and effective area, and thus both sets of experimental data agree reasonably well. The experiments show that above $Re_D\approx 4\times 10^4$ the displacement thickness obtained by Tucker's method agrees with experiment. Below this limit, the experimental displacement thickness increasingly surpasses the prediction. The flow seems to separate at least partially from the nozzle walls for lower Reynolds numbers. In this paper, results for $Re_D>4\times 10^4$ corresponding to $p_0>310$ Torr only are presented. It is noted that in this range δ^* is much smaller than D, i.e., $\delta^*/D\approx 0.07$.

The nozzle exit Mach number determined from static pressure measurements, and confirmed by additional pitot pressure readings was $M=2.88$ for $Re_D>1\times 10^5$, dropping slightly to $M=2.75$ for $Re_D=4\times 10^4$. With the identical nozzle, the Mach number at the beginning of the pressure recovery zone§ could be lowered by lowering the back

‡For more details see Ref. 13.

§For terminology note Fig. 1.

pressure, thus moving the recovery zone downstream in the duct. The average Mach number was then determined combining pressure measurements with a Fanno line (see i.e., Shapiro,[15] p. 160). The range of Mach numbers at the start of the pressure recovery $1.76<M_{\text{start}}<2.88$ was covered. This approach is the same as that of Neumann and Lustwerk.[1] The shifting of the recovery zone in the duct also slightly changes the respective Reynolds number. This parameter, however, was primarily varied by supply pressure changes, giving $4\times10^4<Re_D<4\times10^5$. The boundary-layer parameter δ^*/D at the start of the pressure rise was changed in the range $0.04<\delta^*/D<0.20$ by both the variation of supply pressure and the location of the recovery zone in the duct.

The pressure measurements were made using Statham strain gauge pressure transducers and Wallace and Tiernan dial gauges. These were calibrated with a mercury precision manometer. Pressure was measured to an accuracy of $\pm1\%$ of the required ranges.

Results

For a fixed set of parameters M, Re_D, and δ^*/D at the beginning of the recovery zone, the static pressure distribution along the diffuser typically appears as shown by curve (*a*) in Fig. 4 for the longest duct ($L/D=60$). (To the left of $x/D=0$, i.e., to the left of the nozzle exit, we note the pressure distribution in the nozzle.) As long as a shorter duct remains longer than the recovery length, none of the flow parameters at the beginning of the recovery zone are altered if the back pressure ratio is increased to compensate for the eliminated frictional losses in the removed duct section. This is illustrated by curves (*b*), (*c*), and (*d*) in Fig. 4. Only the extent of the subsonic flow with additional frictional pressure losses beyond the recovery zone is shortened here, whereas the pressure distribution in the recovery zone remains unchanged. Waltrup and Billig[10] previously showed, that the cutoff of the final part of the pressure distribution is unchanged even if the tube length is shorter than the recovery zone. This observation is confirmed by the curves (*d*), (*e*), and (*f*) in Fig. 4.

Figure 5 shows the influence of the Reynolds number Re_d on the pressure recovery. Pressure distributions with fixed M and δ^*/D, but changing Re_D have been shifted in such a way that the start of the recovery zone is matched at $x'/D=0$. A variation of the Reynolds number (based on the hydraulic diameter) at the pressure recovery start by for instance a factor of three, does not alter these curves. In contrast, a variation of only 15% in the boundary-layer parameter δ^*/D leads to readily discernible changes in the pressure distribution. It is thus concluded that within the range of this investigation, the Reynolds number Re_D has little effect on the pressure recovery. This fact greatly simplifies the task of correlating different experiments, since the relevant parameters are reduced to M and δ^*/D.¶

In Fig. 6 we note the results of the dependence of the pressure recovery $\Delta p/p_0$ on the primary parameters M and δ^*/D. This figure is based on the observation that experimental results of $\Delta p/p_0$ at fixed M but variable δ^*/D show a simple linear relationship for $\Delta p/p_0=f(\delta^*/D, M=\text{const})$. Extending such constant Mach number lines to $\delta^*/D=0$, $\Delta p/p_0$ for normal shock pressure recovery is read off. This is an expected result, since it is known that for inviscid flow a normal shock is observed across the entire duct cross section (as reported in experiments with boundary-layer suction, e.g., Shapiro,[15] p. 136). Extending the constant Mach number lines towards large values of δ^*/D, they are all found to intersect the δ^*/D-axis roughly in a single point. Therefore, in drawing Fig. 6 an average value of δ^*/D was chosen for this intersection. However, this value describes a fictitious point., since as expected this experimental data increasingly deviate from the simple behavior for $\delta^*/D>0.1$. At this value the boundary-layer thickness becomes comparable to the channel size and therefore, to the right of the shaded area in Fig. 6, we expect a fully viscous flow in the duct. It should be noted that in the valid range of this correlation $\delta^*<0.1D$, i.e., the boundary-layer displacement thickness is small as compared with the hydraulic diameter.**

The result described in the preceding can be rationalized by noting that a thickening of the boundary layer decreases the effective, central inviscid flow area across which we assume normal shock recovery in a simple model for the pressure increase. Since, as mentioned before, we only deal with very thin displacement thicknesses, an increased displacement

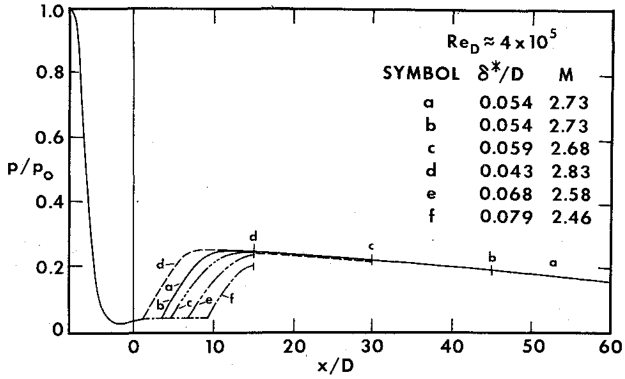

Fig. 4 Pressure distribution measurements in ducts of different lengths: $L/D=15, 30, 45, 60$. The values of δ^*/D and M are given at the start of the recovery zone.

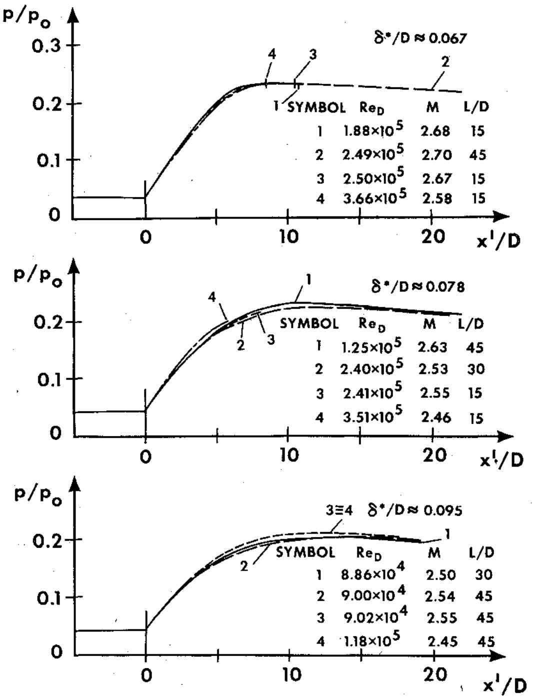

Fig. 5 The influence of the Reynolds number Re_D on the pressure recovery. Shown are sets of pressure distributions at essentially fixed Mach number M and boundary-layer displacement thickness ratio δ^*/D. The pressure profiles have been shifted along x so that the beginning of the recovery zone is superimposed at $x'/D=0$.

¶Waltrup and Billing[10] also show experimental pressure recovery traces. Yet their ducts are generally too short to include the full pressure recovery zone. It is also seen that our traces do not correlate with the highly "parabolic" shapes given by their correlation formula. Too little is known to discuss this empirical result.

**One review of this paper suggested that the fan of lines in Fig. 6 could be reduced to a single line by dividing the values shown on the abscissa by the corresponding ratio for a normal shock. This is indeed true, yet in doing so, information is lost. In using the graph to determine a certain pressure recovery one always would have to look up the normal shock pressure recovery value to obtain the desired result.

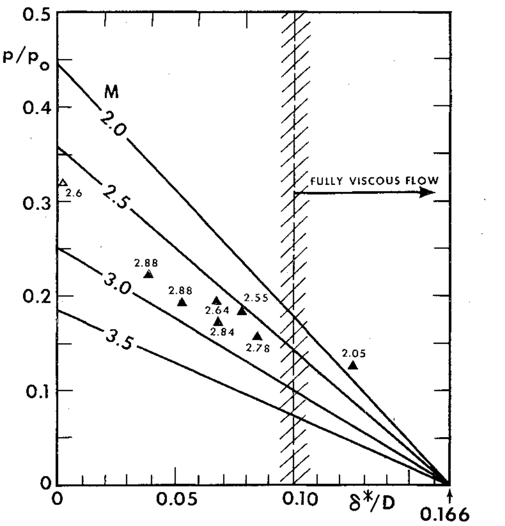

Fig. 6 Pressure recovery $\Delta p/p_0$ as a function of the Mach number M and the boundary-layer parameter δ^*/D. △ Waltrup and Billing.[10] ▲ present work.

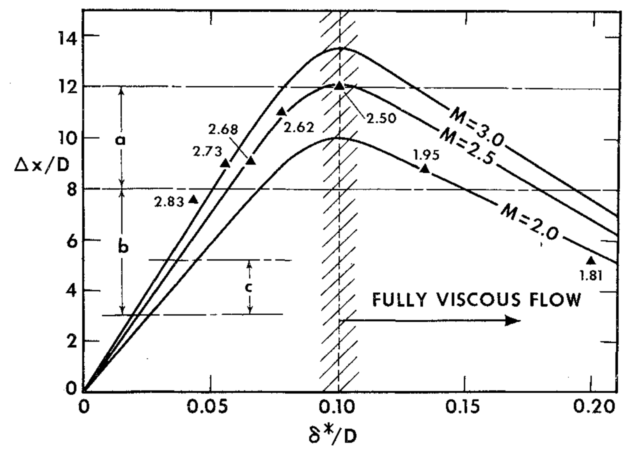

Fig. 7 Dependence of the recovery length of maximum pressure recovery $\Delta x/D$ on the Mach number M and the boundary-layer parameter δ^*/D. a) Experimental range of Neumann and Lustwerk,[1] $1.8<M<4.2$. b) Experimental range of Davidson and Chukhalo,[17] $2<M<5$. c) Experimental range of Waltrup and Billig,[10] $1.5<M<2.7$. ▲ this work.

thickness leads to a practically linearly decreased effective area (e.g., corner effects are negligible). Therefore, a linearly decreased pressure recovery can be expected. This behavior is actually observed in the experiments. Additional frictional pressure losses in the recovery zone can be neglected in this model, since the flow is usually separated from the duct wall in the recovery region due to the high adverse pressure gradients.††

Figure 7 represents what is thought to be a new way of plotting the recovery length, by taking not only M, but also δ^*/D into account. It appears that the second, important parameter has been somewhat neglected in previous studies, excepting the work of Waltrup and Billig[10] and Waltrup and Cameron.[16] Consistent with the ideas discussed so far, a "zero recovery length" for vanishing boundary-layer displacement thickness is expected with the occurrence of a normal shock. The experiments show, that for a fixed Mach number the recovery length increases linearly with δ^*/D until the boundary layers fill the entire tube cross section at $\delta^*/D \sim 0.1$. Earlier work, e.g., Neumann and Lustwerk[1] and Davidson and Chukhalo,[17] confirms the wide range of observed

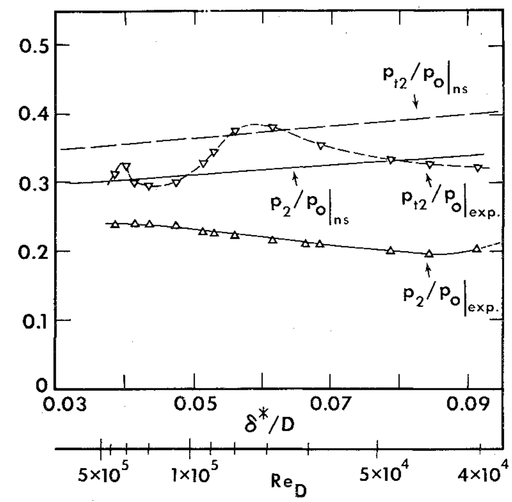

Fig. 8 Static (p_2/p_0) exp. and total pressure (p_{t_2}/p_0)exp. measurements at the exit of the shortest diffuser $(L/D=15)$ at maximum possible back pressure. The pitot pressure is measured in the center of the diffuser exit. The boundary-layer displacement thickness ratio δ^*/D at the nozzle end, being also the start of the recovery zone in this case, is varied by altering the supply pressure i.e., Re_D. The slight effect of Re_D on the nozzle exit Mach number, is reflected by the curves for the static and total pressure normal shock recovery, $(p_2/p_0)_{ns}$ and $(p_{t_2}/p_0)_{ns}$, computed for this Mach number.

recovery lengths at essentially fixed Mach numbers, as shown in Fig. 7. The recovery lengths reported by Waltrup and Billig,[10] where the boundary-layer thickness is known, are in good agreement with this graph. Unfortunately, such comparisons cannot be made with other earlier work due to the lack of information on the boundary layers in most sources.

For the shortest diffuser $(L/D=15)$, Fig. 8 shows the highest possible back pressures (equal to static pressures at the diffuser end) and the corresponding pitot pressure in the center of the exit plane, without affecting the nozzle flow. This means that the back pressure is such that the recovery zone starts immediately at the nozzle end. With $8<\Delta x/D<12$ in our experiments, the recovery length is thus well-contained in the diffuser, see, e.g., curve (d) in Fig. 4. As a reference, the normal shock static and pitot pressure recoveries for the corresponding nozzle exit Mach number are also shown in Fig. 8. Consistent with the previous results, we find the experimental, maximum static pressure at the diffuser exit to increase with reduced displacement thickness. This is obviously not the case for the pitot pressure as measured in the center of the diffuser exit plane, though our diffuser length always exceeds the necessary recovery length for the static pressure. Also, pitot probe traverses show, that the pitot pressure in the exit plane tends to be uniform only for back pressures where the pressure recovery zone moved upstream into the nozzle and the undisturbed nozzle flow has broken down. This observation suggests caution if diffuser pressure recoveries are given, based on pitot pressure measurements only. Due to the nonuniform pitot pressure distributions even well beyond the end of the recovery zone, extensive surveys are necessary to get a reliable mean value. Therefore, it is recommended to use static pressure recovery values for comparisons of diffuser performance. The static pressure is practically constant over the flow cross section and it is easy to measure. Moreover, the static pressure represents the actually achieved pressure recovery.

Conclusions

The Reynolds number based on the flow channel diameter Re_D, the Mach number M, and the boundary-layer thickness ratio δ/D (or displacement thickness ratio, δ^*/D, or $Re_\delta \times \delta/D$ have long been recognized as the governing parameters of pressure recovery in supersonic diffusers. Researchers have

††The complex interaction of shock systems, boundary layers and corner effects are not known and not discussed. At the moment integral statements are feasible only.

given their main attention to the Mach number effects. The Reynolds number dependence is also known in some ranges, especially from comparing results from different facilities with different sizes and thus Reynolds numbers. Yet no previous systematic study on the influence of the boundary-layer thickness on pressure recovery appears to exist in the literature. This situation may be due to the fact that it is more cumbersome to determine δ than M or Re_D, and that in larger wind tunnels the boundary-layer parameters, δ/D or δ^*/D, is usually sufficiently small so that its changes can be neglected. This is no more the case in new diffuser applications. Here we may have narrow flow channels, test section Reynolds numbers below 10^4, and relatively thick boundary-layers. Therefore, Reynolds number and boundary-layer parameter effects must be studied more closely. The present investigation with a rectangular constant area supersonic diffuser following a clean nozzle flow is aimed at this problem. The results show that in narrow flow channels the important parameters in correlating the pressure recovery are the Mach number M and the boundary-layer parameter δ^*/D, rather than the Reynolds number Re_D. The following ranges of these parameters at the beginning of the recovery zone have been covered: $1.76<M<2.88$, $0.04<\delta^*/D<0.20$, $4\times10^4<Re_D<4\times10^5$.

Static and pitot pressure recoveries have been measured. As a result of nonuniform pitot pressure distributions observed, it is suggested that the static pressure recovery should be used in comparing different results and experiments. Here, for the shortest diffuser (i.e., $L/D=15$) and depending on the boundary-layer parameters, the best static pressure recoveries amount to 58-77% of the recovery computed for a normal shock with the same initial Mach number (see Fig. 8)

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